A Simple Modeling Program for Orifice Pulse Tube Coolers

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ABSTRACT

We have developed a calculational model that treats all the components of an orifice pulse tube cooler. We base our analysis on 1-dimensional thermodynamic equations for the regenerator[1] and we assume that all mass flows, pressure oscillations and temperature oscillations are small and sinusoidal. The resulting mass flows and pressures are matched at the boundaries with the other components of the cooler: compressor, aftercooler, cold heat exchanger, pulse tube, hot heat exchanger, orifice and reservoir. The results of the calculation are oscillating pressures, mass flows and enthalpy flows in the main components of the cooler.

By comparing with the calculations of other available models, we show that our model is very similar to REGEN 3 from NIST and DeltaE from Los Alamos National Lab for low amplitudes where there is no turbulence.

Our model is much easier to use than other available models because of its simple graphical interface and the fact that no guesses are required for the operating pressures or mass flows. In addition, the model only requires a minute or so of running time, allowing many parameters to be optimized in a reasonable time.

INTRODUCTION

Pulse Tube coolers are complex systems that require careful optimization of their many components to achieve the best cooling performance. In particular, the regenerator, where a large surface area and high heat capacity are needed to damp out temperature oscillations, is a component that must be designed to maximize heat exchange with the gas passing through it while minimizing the pressure drop across it. The fact that the gas flow is oscillating back and forth while the gas pressure is also oscillating with a different phase makes the analysis of this part of the system too complex for a simple analysis.

We wanted to understand this interaction in the regenerator and, at the same time, we wanted a tool to help us design pulse tube coolers. Toward that end we developed a computer model of the regenerator that would capture the fundamental behavior of the gas-matrix interaction while remaining simple enough to be quick and easy to use. We realized that in optimizing the performance of the regenerator we would be affecting the performance of the other parts of the cooler. Since it is only the net cooling power of the system that is ultimately important, we extended the model to include the other major components of a typical cooler. Now the model (called ARCOPTR for Ames Research Center Orifice Pulse Tube Refrigerator) treats the entire system starting with the compressor and includes the heat exchangers, the regenerator, the pulse tube section itself and also the orifice and reservoir that provide the phase shift of the mass flow that is necessary for cooling to occur.

To simplify the calculations, we limited the model to the consideration of only the fundamental frequency component of the oscillating parameters and we take the limiting case of infinitesimal amplitude oscillations. This allows us to study all the fundamental processes that affect the performance of the pulse tube cooler, but we do not expect highly accurate estimates of performance of actual systems having large amplitudes of pressure oscillation or mass flow. Nevertheless, we feel that the model provides a very useful guide to the optimum trade-off between the many conflicting requirements, especially since higher accuracy is hard to justify without a better understanding of some of the loss mechanisms that occur.

FEATURES OF ARCOPTR

Figure 1 shows the components of an orifice pulse tube cooler that the model treats. Details for the various parts are as follows:

Compressor: The compressor is taken to be adiabatic. All temperature oscillations are assumed to be completely damped in the aftercooler. The equation describing the compressor is:

$$\frac{1}{P_c}\frac{\partial P_c}{\partial t} = -\frac{\gamma}{\zeta_c}\frac{\partial \zeta_c}{\partial t} - \frac{\gamma}{m_c}\frac{\partial m_c}{\partial t} \tag{1}$$

where P_c is the pressure in the compressor, ζ_c is the piston position and m_c is the mass of gas in the compressor.

Regenerator: Here, the 1-D equations developed in a previous paper[1] are used. The basic differential equation in P that resulted is:

$$\begin{aligned} 0 = {} & \frac{\partial^2 P_d}{\partial z^2} + \left[\frac{(1+\alpha i)\,\Gamma T_0 + \alpha^2 + 1}{\left([\Gamma T_0 + 1]^2 + \alpha^2\right) T_0} - \frac{\lambda(\lambda T_0 - i)}{\lambda^2 T_0^2 + 1}\right]\frac{\partial T_0}{\partial z}\frac{\partial P_d}{\partial z} \\ & + \frac{4\pi^2 M^2 (\lambda i T_0 - 1)\left\{(\gamma-1)\left[(1+\alpha i)\,\Gamma T_0 + \alpha^2 + 1\right] - \gamma\left[(\Gamma T_0 + 1)^2 + \alpha^2\right]\right\} P_d}{\varepsilon^2\left[(\Gamma T_0 + 1)^2 + \alpha^2\right] T_0} \end{aligned} \tag{2}$$

where P_d is the dynamic (oscillating) pressure amplitude (scaled by the average pressure), $\alpha = \frac{2\pi(1-f)\,\rho_m^* C_m r_h}{f \tau^* h}$, $\Gamma = \frac{(1-f)\,\rho_m^* C_m}{f \rho_0^* C_p}$, $\lambda = f \mu \tau^* / 2\pi\ K_p \rho_0^*$, T_0 is the

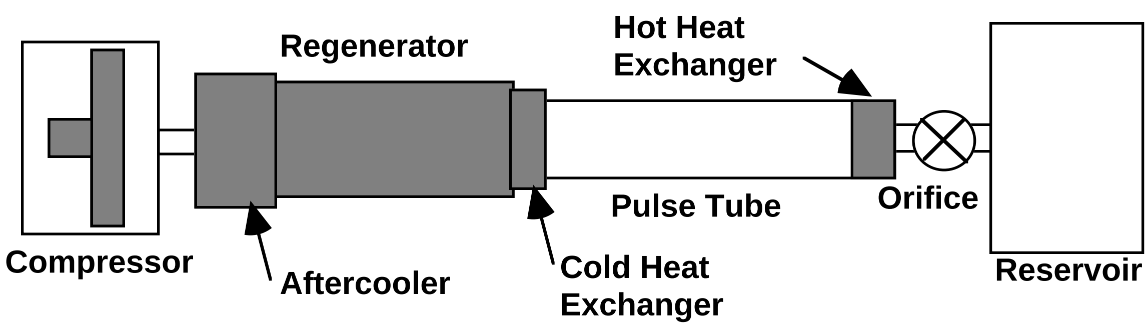

Figure 1. Main components of an orifice pulse tube cooler.

steady component of the local temperature, M = Mach number = $q_0^*/\sqrt{\gamma R T_0^*}$, q_0^* is the effective velocity of the gas at the regenerator inlet, $\gamma = C_p/C_v$, R is the gas constant, f is the void fraction, $\varepsilon = \tau^* q_0^*/L_0^*$, μ is the viscosity, r_h is the hydraulic radius of the matrix, τ^* is the period of oscillation, L_0^* is the length of the regenerator, h is the heat transfer coefficient, ρ_m^* is the matrix density, C_m is the matrix heat capacity, ρ_0^* is the gas density at the warm end, and K_p is the Darcy permeability of the matrix (from the friction factor—see below).

Equation (2) is solved numerically to arrive at a fundamental-frequency amplitude for the pressure; temperature and velocity are then derived from this pressure solution. An initial linear temperature gradient from the hot end to the cold end is used to estimate temperature-dependent values of μ, h and α. The heat flow to the solid matrix is taken to be in phase with the temperature difference between the gas and matrix. This assumes that the dimensions of the pores containing the gas are small enough that the gas is in fairly good contact with the matrix; in other words, the hydraulic radius of the pores is less than the diffusion length in the gas.

The parameters for heat transfer to the matrix and for friction factor come from Kays and London[2]. Since these data show considerable non-linearities at higher velocities, it was felt important to include some high-velocity effects. An initial guess to the velocity in the regenerator is used to estimate the heat transfer coefficient and the friction factor; the regenerator equations are then solved and much more accurate values of the velocity are found. From these velocities the final values for the heat transfer coefficient and the friction factor are found and the calculation is repeated. This is the only instance where some allowance for finite-velocity effects is included. In calculating the effect of thermal conduction axially through the metal matrix, a correction factor of 0.3 is used to reflect the poor contact between adjacent screens.

The boundary conditions for the regenerator solution are that the pressures and mass flows at the ends of the regenerator match those of the compressor and pulse tube (as modified by the aftercooler and cold heat exchanger, respectively).

Heat Exchangers: The aftercooler and the cold and hot heat exchangers are assumed to be isothermal. The primary effect they have on the modeling is to introduce a pressure drop due to their impedance and a phase shift in the mass flow due to their void volume. No attempt is made to assess their adequacy for heat transfer or to calculate the amount of heat that flows through them. It is assumed that all temperature oscillations are completely damped in passing through them. The equation describing the flow in the heat exchangers is just Eq. (2) with the second term on the right missing, since $\partial T/\partial z = 0$:

$$0 = \frac{\partial^2 P_d}{\partial z^2} + \frac{4\pi^2 M^2 (\lambda i T_0 - 1)\left\{(\gamma-1)\left[(1+\alpha i)\Gamma T_0 + \alpha^2 + 1\right] - \gamma\left[(\Gamma T_0 + 1)^2 + \alpha^2\right]\right\} P_d}{\varepsilon^2 \left[(\Gamma T_0 + 1)^2 + \alpha^2\right] T_0} \quad (3)$$

where λ, Γ and α take on appropriate values for the heat exchanger being considered. Since T_0 is independent of z, the coefficient of P_d is just a constant; this equation is solved analytically.

Pulse Tube: In the pulse tube there is no pressure gradient. The mass conservation and the energy conservation equations can be used to arrive at an equation for the effective velocity, q_d:

$$\frac{\partial}{\partial z} q_d - \frac{\Gamma_{pt} q_d}{\Gamma_{pt} T_a + 1 + \alpha_{pt} i}\left(\frac{T_{hot} - T_{cold}}{L_{pt}}\right) = \frac{2\pi f_{pt}\left(\gamma \Gamma_{pt} T_a + 1 + \alpha_{pt} i\right) P_d}{\gamma i \left(\Gamma_{pt} T_a + 1 + \alpha_{pt} i\right)} \quad (4)$$

where T_a is the z-dependent steady temperature and L_{pt} is the length of the pulse tube (scaled by the length of the regenerator). Γ_{pt} and α_{pt} in this equation are defined in the same way as Γ and α for the regenerator, above, where heat exchange with the pulse tube wall has replaced the heat exchange with the regenerator matrix. This equation can be solved analytically if it is assumed that T_a is linear in z and that α_{pt} and Γ_{pt} are independent of z.

However, the assumption, made for the regenerator, that the heat transfer to the wall is in phase with the temperature difference between the gas and the wall is no longer adequate since

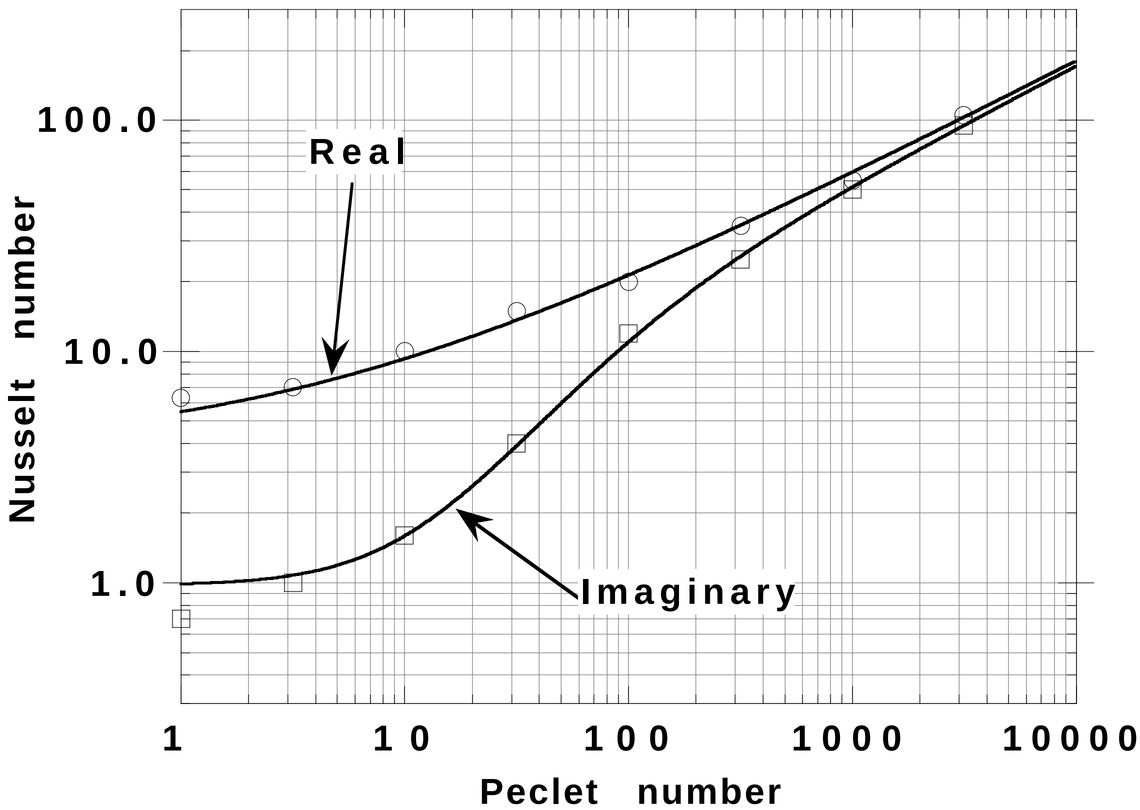

Figure 2. The real and imaginary Nusselt numbers from the data of Kornhauser.

the gas in the center of the pulse tube can be many diffusion lengths from the wall. When that is the case it is possible to have the heat transfer occurring at a phase different from that of the phase of δT between the wall and the average temperature of the gas. This leads to the concept of a complex Nusselt number as discussed by Kornhauser[3]. The effect on Eq. (4) is to make α_{pt} complex since $h = k\ Nu/r_h$ where k is the thermal conductivity of the gas, Nu is the complex Nusselt number and r_h is the hydraulic radius of the pulse tube (r_h = cylinder volume/cylinder surface = $0.5\ r_{pt}$). A typical result from Kornhauser's data is shown in Fig. 2. The Peclet number used here is $Pe = \rho\ C_p\ \omega\ r_h^2 / 4\ k$, which is just $(r_h / 2L_d)^2$ where L_d is the diffusion length in the gas. The curves we fit to the above data are:

$$Re\{Nu\} = 5.603 + 1.170\sqrt{(Pe)} \quad \text{and} \quad Im\{Nu\} = \frac{(16.43 + 1.140\ Pe^{1.5})}{25.02 + Pe} \tag{5}$$

Orifice and Reservoir: In a pulse tube section with no wall interaction and no flow out the orifice, all the mass flow into the cold end goes into compressing the gas in the pulse tube. This mass flow will be 90° out of phase with both the pressure and the temperature and there will be no work flow or enthalpy flow at the cold end. If there is flow out the orifice, this will add a component of flow throughout the pulse tube that is in phase with the pressure and leads to an enthalpy flow that is the basis for the cooling in an orifice pulse tube cooler. If the orifice flow is too large, however, the pressure drop in the regenerator will yield a very small pressure oscillation in the pulse tube, and result in very little cooling. Clearly, there is an optimum orifice setting that produces the best cooling.

Our model treats the orifice as an impedance with flow proportional to pressure; the flow is symmetrical with flow direction and has no dependence on velocity. The reservoir is assumed to be an infinite volume as far as the interaction with the rest of the system is concerned. For convenience, the mass flow into the reservoir is expressed in terms of a pressure oscillation in a specified finite volume; for correct results, the reservoir volume must be big enough to make these pressure oscillations negligible compared to those in the rest of the system.

Solution of the model. Equation (2) for the regenerator is solved numerically. We have two versions of the model calculation; they each solve the equation by a different technique. One version uses a 'shooting' method where the slope of the pressure at the warm end of the regenerator is adjusted until the pressure and mass flow at the cold end, as determined by integrating the differential equation, agree with the boundary conditions. The other method is a two-point-boundary-condition technique that guesses a parabolic pressure profile for the regenerator that satisfies the boundary conditions but doesn't satisfy eq. (2). This pressure profile is then iteratively adjusted until it satisfies eq. (2). These two very different methods give identical results.

The analytic solutions for the compressor and for the pulse tube provide relationships between pressure and mass flow at each end of the regenerator; these relationships (rather than fixed values of pressure or flow) make up the boundary conditions at each end of the regenerator for the solution of eq. (2). The analytic solutions for the heat exchangers modify these boundary conditions in a way that accounts for the pressure drop and the phase shift in the mass flow that occurs in the heat exchangers. The general form of the relationship between pressure and mass flow is not changed by the heat exchangers.

COMPARISON WITH OTHER MODELS

The Modeled Cooler

For the comparisons, a pulse tube cooler of the following dimensions was modeled:
Aftercooler: L = 1.25 cm, I. D. = 3.62 cm, mesh = # 87, wire diam. = 0.0115 cm.
Regenerator: L = 5.0 cm, I. D. = 1.22 cm, mesh = (see tables), wall thickness = 0.05 cm.
Cold heat exchanger: L = 0.2 cm, I. D. = 1.128 cm, mesh = # 87, wire diam. = 0.0115 cm.
Pulse tube: L = 3.0 cm, I. D. = 1.128 cm, wall thickness = 0.0085 cm.
Hot heat exchanger: L = 0.5 cm, I. D. = 1.128 cm, mesh = # 87, wire diam. = 0.0115 cm.
Orifice setting: 0.247 g/s bar.
Reservoir volume: 500 cm^3.
System pressure: Helium at 20 bar.
Hot temperature: 300 K.
Cold Temperature: 80K.
Operating frequency: 55 Hz.

The aftercooler, cold heat exchanger and hot heat exchanger are copper, the regenerator (including screens) and the pulse tube are stainless steel. The screens in the regenerator have a void fraction of 0.69.

The Comparison Models:

REGEN3[4] from NIST is a model of the regenerator only. It treats the full time-dependence of the oscillating parameters, so it can capture distortions to the waveforms. It handles large amplitude oscillations. It assumes a small pressure drop in the regenerator so it might not be suitable for very restrictive regenerators. The other parts of the cooler such as the compressor, the pulse tube and the orifice must be modeled by other means.

DeltaE[5] from Los Alamos National Laboratory models an entire cooler (except the compressor). Like ARCOPTR it is a linearized model that treats only the lowest order sinusoidal component of the oscillations. It is based on thermo-acoustic wave equations so it is especially suitable for higher-frequency systems where the dimensions are comparable to an acoustic wavelength. It can treat regenerators with large pressure drops and it uses correlations for heat flow and friction factor that include nonlinear effects that occur at high velocities.

Results of the Comparison

For the ARCOPTR calculation, the compressor stroke was adjusted to give the desired inlet

Table 1. Model Comparison for P_{in} = 0.600 bar and # 400 Mesh Screens.

	REGEN3 (NIST)	**DeltaE** (LANL)	**ARCOPTR** (NASA Ames)
Input			
Regenerator mesh = # 400 wire = 0.0025 cm	$\dot{m}_{in}$= 0.497 @ 30.2° $\dot{m}_{out}$= 0.411 @ 13.2°	P_{in} = 0.600 bar @ 0°	P_{in} = 0.600 bar @ 0°
Output			
Compressor PV work (W)	4.32	4.36	4.03
$\dot{m}$ into regenerator (g/s)	(input)	0.492 @ 29.7°	0.497 @ 30.2°
$\dot{m}$ out of regenerator (g/s)	(input)	0.415 @ 13.3°	0.411 @ 13.2°
ΔP across regenerator (bar)	0.215	0.242 @ 25.1°	0.227 @ 25.6°
Regenerator energy flow (W) (gas enthalpy + matrix conduct.)	0.95	1.00	1.26
Pulse tube pressure (bar)	(not available)	0.394 @ –15.2°	0.408 @ –13.9°
Pulse tube enthalpy flow (W)	(not available)	0.45	0.49

pressure into the regenerator. For DeltaE, the inlet pressure is an input parameter. For REGEN3, mass flows in and out of the regenerator are the starting parameters. The energy flows quoted for REGEN3 are not exact because the calculation did not completely converge to give a consistent value of total energy flow (enthalpy flow in the gas plus conduction in the matrix) at the two ends of the regenerator in runs lasting overnight. Note: all of the pressure and mass flow values in the table are the amplitudes of oscillating variables; peak-to-peak values would be twice the quoted values. Enthalpy flows and PV work are time-averaged steady values.

Table 1 shows the comparison for the regenerator filled with # 400 mesh screens when the warm-end pressure oscillations are ±0.6 bar for a 20 bar ambient pressure. The ARCOPTR results agree within 7% of both REGEN3 and DeltaE for ΔP and within 1% of DeltaE on the mass flows in the regenerator. For the energy flow (loss) in the regenerator, ARCOPTR is about 30% higher than the other models, while the compressor PV work is 8% lower than the others. In the pulse tube, ARCOPTR has 10% more enthalpy flow (cooling power) and 4% more pressure oscillation than DeltaE.

Table 2 shows the comparison for the regenerator filled with # 400 mesh screens when the

Table 2. Model Comparison for P_{in} = 2.400 bar and # 400 Mesh Screens.

	REGEN3 (NIST)	**DeltaE** (LANL)	**ARCOPTR** (NASA Ames)
Input			
Regenerator mesh = # 400 wire = 0.0025 cm	$\dot{m}_{in}$= 1.75 @ 28.4° $\dot{m}_{out}$= 1.43 @ 9.0°	P_{in} = 2.400 bar @ 0°	P_{in} = 2.400 bar @ 0°
Output			
Compressor PV work (W)	64.5	60.1	57.5
$\dot{m}$ into regenerator (g/s)	(input)	1.71 @ 27.9°	1.75 @ 28.4°
$\dot{m}$ out of regenerator (g/s)	(input)	1.42 @ 8.9°	1.43 @ 9.0°
ΔP across regenerator (bar)	1.159	1.208 @ 21.5°	1.136 @ 23.0°
Regenerator energy flow (W) (gas enthalpy + matrix conduct.)	3.50	3.20	3.84
Pulse tube pressure (bar)	(not available)	1.352 @ –19.9°	1.424 @ –18.1°
Pulse tube enthalpy flow (W)	(not available)	6.15	6.03

Table 3. Model Comparison for P_{in} = 0.600 bar and # 200 Mesh Screens.

	REGEN3 (NIST)	**DeltaE** (LANL)	**ARCOPTR** (NASA Ames)
Input			
Regenerator mesh = # 200 wire = 0.0025 cm	$\dot{m}_{in}$ = 0.626 @ 33.9° $\dot{m}_{out}$ = 0.527 @ 22.0°	P_{in} = 0.600 bar @ 0°	P_{in} = 0.600 bar @ 0°
Output			
Compressor PV work (W)	4.82	5.24	4.87
$\dot{m}$ into regenerator (g/s)	(input)	0.638 @ 34.7°	0.626 @ 33.9°
$\dot{m}$ out of regenerator (g/s)	(input)	0.543 @ 22.6°	0.527 @ 22.0°
ΔP across regenerator (bar)	0.096	0.101 @ 31.8°	0.092 @ 31.0°
Regenerator energy flow (W) (gas enthalpy + matrix conduct.)	1.95	2.10	2.55
Pulse tube pressure (bar)	(not available)	0.516 @ –5.9°	0.523 @ –5.2°
Pulse tube enthalpy flow (W)	(not available)	0.83	0.81

warm-end pressure oscillations are ±2.4 bar for a 20 bar ambient pressure. The ARCOPTR results agree within 6% of both REGEN3 and DeltaE for ΔP and within 2% of DeltaE on the mass flows in the regenerator. For the energy flow (loss) in the regenerator, ARCOPTR is 20% higher than DeltaE and 10% higher than REGEN3, while the compressor PV work is 5% lower than DeltaE and 11% lower than REGEN3. In the pulse tube, ARCOPTR has 2% less enthalpy flow (cooling power) and 5% more pressure oscillation than DeltaE.

Table 3 shows the comparison for the regenerator filled with # 200 mesh screens when the warm-end pressure oscillations are ±0.6 bar for a 20 bar ambient pressure. The ARCOPTR results agree within 9% of both REGEN3 and DeltaE for ΔP and within 3% of DeltaE on the mass flows in the regenerator. For the energy flow (loss) in the regenerator, ARCOPTR is 21% higher than DeltaE and 31% higher than REGEN3, while the compressor PV work is 7% lower than DeltaE and 1% higher than REGEN3. In the pulse tube, ARCOPTR has 3% less enthalpy flow (cooling power) and 1% more pressure oscillation than DeltaE.

Summary. ARCOPTR results agree well with REGEN3 and DeltaE for all three cases studied. Pressure drops, mass flows and compressor PV work of ARCOPTR are within about 10% of the other models. The energy flow (loss) term for the regenerator is 12-30% higher than the other models, while the pulse tube enthalpy flow (cooling power) is within 10% of that of DeltaE. It is not clear why ARCOPTR is consistently higher than the other models for the regenerator energy flow.

CONCLUSIONS

Our model treats all the components of an orifice pulse tube cooler using rigorous 1-D thermodynamic equations. It linearizes the equations, looking only at the fundamental frequency terms in the limit of small amplitudes. Since most of the important phenomena in a pulse tube cooler should be evident in such a treatment, the model should have great utility in suggesting ways to optimize performance of an actual cooler. It isn't clear that the use of higher harmonics to describe the waveforms is a great advantage when there are important loss mechanisms due to 2-D flow effects in the pulse tube section which are not being treated yet.

Comparing our model with REGEN3 from NIST and DeltaE from LANL, we find quite good agreement for the three cases studied. In addition, we have found that our model is much faster and easier to use than the other two.

We are currently looking at the effects of convection and other 2-D phenomena[6] in the pulse tube section and we plan to incorporate these results into the model when they become available. We feel that this should greatly improve the ability of the model to describe real-world coolers.

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